Math 131B-1: Homework 8

Due: March 7, 2014

1. Read Tao Sections 16.1-4.
2. Do Tao exercises 15.7.10, 16.2.2.
3. Do Tao exercise 16.2.3 [Hint: It's important to pick a function g for which $||g||_\infty \neq \int_0^1 g$. Try working with g such that $g(x) = x^2 - x$ on $[0, 1]$ and g is extended periodically to the rest of the real line.]
4. Do Tao exercise 16.2.6. [Hint: Remember that a continuous 1-periodic function f is the same as a continuous function on $[0, 1]$ with $f(0) = f(1)$. So you only have to define all your sequences on the interval in this problem.]
5. Prove Pythagoras' Identity: If $< f, g >= 0$, then $||f + g||_2^2 = ||f||_2^2 + ||g||_2^2$.
6. Prove that the convolution $f * g$ of two continuous $\mathbb{Z}$-periodic function is continuous. [Hint: We know $|f(x)| < M$ for some $M > 0$. So start by deciding that $|f * g(x) - f * g(x')| = |\int_0^1 f(y)g(x-y)dy - \int_0^1 f(y)g(x'-y)dy| \leq M|\int_0^1 (g(x-y) - g(x'-y))dx$. Now use uniform continuity of g.]

   [Note: This may seem short, but two of the problems above have several parts.]